Pre-Sixth Form Mathematics
Summer Self-Study Pack

Mathematics Department
RGS High Wycombe

Introduction

This booklet has been designed to aid the transition of students who intend on studying Mathematics when entering the Sixth Form. Many areas of AS and A2 Mathematics require the confident use of algebraic and numerical concepts. Our experience has taught us that without such confidence, many students will struggle and perhaps not make the progress that they should.

It is hoped that the completion of this booklet will provide several benefits:

- The student's new teachers will obtain an overview of their areas of strength and weakness;
- Mathematical skill levels will be maintained during the summer holiday;
- Students will consolidate and perhaps enhance their knowledge of key mathematical concepts.

PLEASE NOTE: At the back of this booklet you will find some exercises that need to be completed before September.

- **All sections should be attempted.**
- **All work should be completed on A4 lined paper, with your name clearly written at the top.**
- **Solutions need to be well-structured and presented in a neat, orderly manner.**
- **The final answers to the exercise questions appear at the back of the booklet; consequently, it is expected that students will produce full solutions and show all workings.**
- **It is also expected that students tick solutions that they complete successfully, thus allowing their teachers to focus on the areas of difficulty.**
- **There are MyMaths references to provide you with extra questions to test yourself and lesson notes to help you learn any parts you were not sure about when studying for your Mathematics GCSE.**
- **The work is to be completed before your test at the beginning of September. However, since the password will change for MyMaths at the end of the holiday, it is better to attempt the assessment questions as soon as possible so that you are aware of your own strengths and areas for development and have time to practise these skills. You will be offered help next term.**

THE COMPLETION OF THIS BOOKLET IS A REQUIREMENT FOR STUDYING A-LEVEL MATHEMATICS AT THE RGS.

There are notes and examples contained within this booklet to help you. Some of the exercises are demanding. Do not panic. Persevere. You will feel the benefits later in the course.

Have fun!

A: Expanding brackets

We often need to expand (multiply out) brackets in order to simplify an expression. Various methods may be employed, many people use the FOIL method (shown below), but the key point to remember is that everything on the inside needs to be multiplied by everything on the outside.

- $x(2x+3y^2)=2x^2+3xy^2$

- Expand $(x+2)(x-3)$ using the FOIL method (First, Outside, Inside, Last)

$$(x+2)(x-3)=x^2-3x+2x-6=x^2-x-6$$

With care, we can expand brackets containing any number of terms,

$$\begin{aligned}(3a-b-c)(a+2b+3c) &= 3a(a+2b+3c)-b(a+2b+3c)-c(a+2b+3c)\\ &= 3a^2+6ab+9ac-ab-2b^2-3bc-ac-2bc-3c^2\\ &= 3a^2-2b^2-3c^2+5ab-5bc+8ac\end{aligned}$$

Certain results are important and it is worth the effort to learn them…

- $(a+b)^2=a^2+ab+ab+b^2=a^2+2ab+b^2$

- $(a-b)^2=a^2-ab-ab+b^2=a^2-2ab+b^2$

- $(a+b)(a-b)=a^2-ab+ab-b^2=a^2-b^2$ [This is known as the difference of two squares]

Therefore,

- $(3x-2y)^2=(3x)^2-2(3x)(2y)+(2y)^2=9x^2-12xy+4y^2$

- $(3x-2y)(3x+2y)=(3x)^2-(2y)^2=9x^2-4y^2$

MyMaths Reference: Algebra → Algebraic manipulation → Brackets

B: Factorising expressions

Whilst it is important to be able to expand brackets, it is possibly more important to be able to reverse the process; that is, to be able to factorise an expression.

Common factors

Some expressions can be factorised by identifying common factors.

- $3xy - 12x^2 = 3(xy - 4x^2) = 3x(y - 4x)$ [this expression has two common factors, 3 and *x*]

Four-term expressions

Some expressions can be factorised by grouping in pairs.

- $2ax + 3ay - 4bx - 6by = a(2x + 3y) - 2b(2x + 3y) = (a - 2b)(2x + 3y)$

$(2x + 3y)$ is now a common factor

Quadratics

Depending on the particular quadratic, the process of factorisation may be easy or difficult.

Using the difference of two squares

Be on the look out for these situations,

- $x^2 - 4y^2 = (x)^2 - (2y)^2 = (x + 2y)(x - 2y)$
- $8x^2 - 50 = 2(4x^2 - 25) = 2(2x + 5)(2x - 5)$

When the coefficient of x^2 is one

Simply find two numbers that multiply to give the constant and sum to give the coefficient of *x*

- $x^2 + x - 6 = (x + 3)(x - 2)$ multiply to give –6 and add to give +1; i.e. 3 and –2

MyMaths Reference: Algebra → Algebraic manipulation → Factorising Quadratics 1

When the coefficient of x^2 is not one

This is more difficult. For example, if we needed to factorise $4x^2 - 4x - 15$, the solution could be of the form (4*x* + ?)(*x* + ??) or (2*x* + ?)(2*x* + ??). If you are lucky you might be able to spot the correct factorisation, but most people would have to resort to the following algorithm.

1. Multiply the coefficient of x^2 by the constant term — 4 × – 15 = – 60
2. Find factors of – 60 that sum to give the coefficient of x (i.e. – 4) — + 6 – 10 = – 4
3. Split the middle term using these numbers — $4x^2 + 6x - 10x - 15$
4. Factorise the first two terms and then the last two terms — $2x(2x + 3) - 5(2x + 3)$
5. Complete the factorisation… easy! — $(2x - 5)(2x + 3)$

MyMaths Reference: Algebra → Algebraic manipulation → Factorising Quadratics 2

C: Quadratic equations

The ability to calculate the roots of a quadratic equation is extremely useful. Quadratic equations occur in the most unlikely areas of mathematics – the flight of a projectile, for example.

Please note that the problem may require you to rearrange an equation into the form $ax^2 + bx + c = 0$ before attempting to solve it.

$$x - 67 + 3x^2 = 9 + 6x - 2x^2$$
$$5x^2 - 5x - 76 = 0$$

Factorisation

We can use factorisation (see above for details) to solve $ax^2 + bx + c = 0$.

Example: Solve $x^2 + x - 6 = 0$

- As we have seen above, $x^2 + x - 6 = 0$

  $$(x+3)(x-2) = 0$$

- Now if the left-hand side is equal to zero, either $(x + 3) = 0$ or $(x - 2) = 0$
- Therefore, the roots of the equations are $x = -3$ and $x = 2$.

- The solution to the equation is the set {–3,2}; i.e. all roots to the equation.

Example: Solve $4x^2 - 4x - 15 = 0$

- As we have seen above, $4x^2 - 4x - 15 = 0$

  $$(2x-5)(2x+3) = 0$$

- Employing the same logic as before, we see that the roots are $x = 2\frac{1}{2}$ and $x = -1\frac{1}{2}$

MyMaths Reference: Algebra → Equations - quadratic → Quadratics equations

The quadratic formula

This can be used to solve quadratic equations by inputting the coefficients of $ax^2 + bx + c = 0$ into the following equation:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example:

Solve $x^2 + 3x - 2 = 0$

Here a = 1, b = 3 and c = – 2

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-3 \pm \sqrt{3^2 - (4 \times 1 \times -2)}}{2 \times 1}$$

$$x = \frac{-3 \pm \sqrt{17}}{2}$$

$x = 0.562$ or $x = -3.56$ (3sf)

MyMaths Reference: Algebra → Equations - quadratic → Quadratic formula

D: Manipulating formulae

Single occurrence

- Make a the subject of $s = ut + \frac{1}{2}at^2$

$$\frac{1}{2}at^2 = s - ut$$ [a only occurs in one term $\therefore$ isolate it]

$$at^2 = 2(s - ut)$$

$$a = \frac{2(s - ut)}{t^2}$$

- Make h the subject of $S = \pi r\sqrt{h^2 + r^2}$

$$\frac{S}{\pi r} = \sqrt{h^2 + r^2}$$ [isolate the square root]

$$\left(\frac{S}{\pi r}\right)^2 = h^2 + r^2$$ [square both sides]

$$h^2 = \left(\frac{S}{\pi r}\right)^2 - r^2$$

$$h = \sqrt{\left(\frac{S}{\pi r}\right)^2 - r^2}$$

MyMaths Reference: Algebra → Expressions and formulae → Rearranging 1

Multiple occurrences

With multiple occurrences, collect all occurrences of the relevant variable on one side of the equation and factorise.

- Make x the subject of $y = \frac{x+1}{x+2}$

$$y = \frac{x+1}{x+2}$$

$$y(x+2) = x+1$$

$$xy + 2y = x+1$$

$$xy - x = 1 - 2y$$ ← All the xs are on one side; now we can factorise…

$$x(y-1) = 1 - 2y$$

$$x = \frac{1-2y}{y-1}$$

MyMaths Reference: Algebra → Expressions and formulae → Rearranging 2

E: Indices

Definitions

In a^m, a is the base and m is the index. Please note that the plural of index is indices, not indicies.

Index laws

If two quantities are in the same base then the following rules apply:

$$a^m \times a^n = a^{m+n}$$
$$a^m \div a^n = a^{m-n}$$
$$(a^m)^n = a^{mn}$$
$$a^0 = 1$$
$$a^{-m} = \frac{1}{a^m}$$
$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

Do not confuse these two rules… ($a^m \times a^n = a^{m+n}$ and $(a^m)^n = a^{mn}$)

Questions may require you to convert all quantities to the same base and/or combine several of the rules above.

Examples:

Find the value of:

a) $81^{\frac{1}{2}}$ $\quad 81^{\frac{1}{2}} = \sqrt{81} = 9$

b) $81^{\frac{3}{4}}$ $\quad 81^{\frac{3}{4}} = \left(\sqrt[4]{81}\right)^3 = (3)^3 = 27$

c) $16^{-\frac{3}{4}}$

$$16^{-\frac{3}{4}} = \frac{1}{16^{\frac{3}{4}}}$$
$$= \frac{1}{\sqrt[4]{16^3}} = \frac{1}{\left(\sqrt[4]{16}\right)^3}$$
$$= \frac{1}{2^3}$$
$$= \frac{1}{8}$$

We can either cube 16 and then find the fourth-root; or we can find the fourth-root of 16 and cube the answer. Obviously, one option is much easier than the other.

MyMaths Reference: Number → Powers and roots → Indices 1

MyMaths Reference: Number → Powers and roots → Indices 2

MyMaths Reference: Number → Powers and roots → Indices 3

F: Completing the square

The process of completing the square involves re-writing (ax^2+bx+c) as $a(x+p)^2+q$; that is, a 'square' plus an 'adjustment'.

When a = 1

For example, let's put the quadratic equation $x^2-4x-3=0$ into completed square form.

- Clearly, if we wish to end up with x^2-4x, we need to begin with $(x-2)^2$
- $(x-2)^2=x^2-4x+4$, which is nearly the quadratic required
- However, we don't want '+ 4', we want '– 3' and so we must subtract 7: this is the 'adjustment'.

$$x^2-4x-3=(x-2)^2-7$$

If the coefficient of x^2 is one then the number in the bracket is half of the coefficient of x

- Complete the square for x^2+6x+1

$$x^2+6x+1=0$$
$$(x+3)^2-9+1=0$$
$$(x+3)^2-8=0$$

When a ≠ 1

In the case where a ≠ 1, we start by taking a out as a factor; then we complete the square for the quadratic inside the bracket; before finally multiplying out.

Example:

Complete the square for $2x^2-x+1$

The first step is to take out the coefficient of x^2 as a factor of the first two terms

$$2\left[x^2-\frac{1}{2}x\right]+1$$

Now we complete the square as before

$$=2\left[\left(x-\frac{1}{4}\right)^2-\frac{1}{16}\right]+1$$

$$=2\left(x-\frac{1}{4}\right)^2-\frac{2}{16}+1$$

$$=2\left(x-\frac{1}{4}\right)^2+\frac{7}{8}$$

As before, this number is half of the coefficient of *x*, i.e. half of $-\frac{1}{2}$

For the adjustment, subtract the square of the number in the bracket

Multiply out the bracket, including the adjustment, then simplify

MyMaths Reference: Algebra → Equations - quadratic → Completing the square

G: Algebraic fractions

Algebraic fractions may be dealt with in much the same way as numerical fractions.
The key points to remember are:

- Factorise all numerators and denominators before proceeding; be on the look out for quadratics that can be factorised using the difference of two squares;
- Once you have factorised, cancel any factors which appear in the numerator and denominator
- For addition / subtraction, find the lowest common multiple (LCM) for the denominator;

Examples:

- $\frac{4x}{6x^2-2x}=\frac{4x}{2x(3x-1)}=\frac{2}{3x-1}$ ← Cancel a factor of $2x$

- $\frac{2x^2-x-1}{4x^2-1}=\frac{(2x+1)(x-1)}{(2x+1)(2x-1)}=\frac{x-1}{2x-1}$ ← Difference of two squares

- $\frac{3}{7x}+\frac{2x+1}{7x}=\frac{2x+4}{7x}$ ← Same denominators, so just add numerators

- $\frac{7}{4x}-\frac{3x-1}{2x}=\frac{7}{4x}-\frac{6x-2}{4x}=\frac{7-(6x-2)}{4x}=\frac{9-6x}{4x}$

  Write the fractions with a common denominator (in this case by doubling the top and bottom of the second fraction)

  Be careful when subtracting something involving negatives – use brackets to help

- $\frac{16x}{25y}\times\frac{5y^3}{4x^2}=\frac{4y^2}{5x}$ ← Cancel the numbers first, eg 16 and 4, 25 and 5. Then cancel each letter in turn, x's then y's.

- $\frac{8x}{3y^3}\div\frac{14x^2}{9y^4}=\frac{8x}{3y^3}\times\frac{9y^4}{14x^2}=\frac{4}{1}\times\frac{3y}{7x}=\frac{12y}{7x}$ ← Remember to flip the second fraction and multiply.

MyMaths Reference: Algebra → Algebraic manipulation → Cancelling algebraic fractions

MyMaths Reference: Algebra → Algebraic manipulation → Adding algebraic fractions

MyMaths Reference: Algebra → Algebraic manipulation → Multiplying algebraic fractions

H: <u>Simultaneous equations</u>

A linear equation in the form y = mx + c represents a straight line with gradient m and y-intercept (0, c).

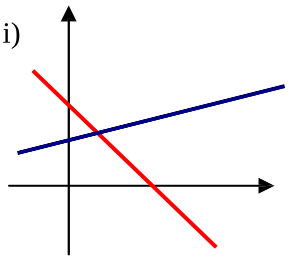
If we have two such linear equations, which are simultaneously true, then there are three possible outcomes.

i). There may be a unique solution; the lines intersect at one point.

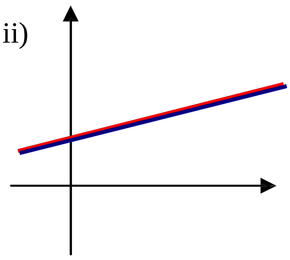
ii). There may be an infinite number of solutions; both equations refer to the same line.

iii). There is no solution; the lines are parallel.

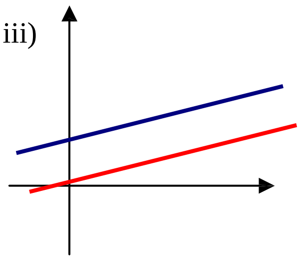
i)



ii)



iii)



A simultaneous equation may be solved by either:

Elimination

- Multiply one equation to ensure that you have the same number of *x* or *y* in each equation, then add or subtract as required.
- **<u>Example</u>**: Solve the simultaneous equation 3x + 2y = 4 and 2x + 5y = – 1

$$
\begin{array}{lll}
 & 3x+2y=4 & (A) \\
 & 2x+5y=-1 & (B) \\
 & & \\
2\times(A) & 6x+4y=8 & (C) \\
3\times(B) & 6x+15y=-3 & (D) \\
 & & \\
(D)-(C) & 11y=-11 & \\
 & & \\
 & y=-1 & \\
 & & \\
\text{Sub into } (A) & 3x-2=4 & \\
 & x=2 &
\end{array}
$$

MyMaths Reference: Algebra → Equations simultaneous → Simultaneous equations 2

MyMaths Reference: Algebra → Equations simultaneous → Simultaneous equations 3

MyMaths Reference: Algebra → Equations simultaneous → Negative equations

Substitution

- Rearrange one of the equations to make either x or y the subject, then substitute this expression into the other equation.

Example: Solve the simultaneous equation $3x + 2y = 4$ and $2x + y = 3$

$y = 3 - 2x$	Rearrange second equation
$3x + 2(3 - 2x) = 4$ $3x + 6 - 4x = 4$ $x = 2$	Sub expression for y into first equation
$6 + 2y = 4$ $y = -1$	Sub into first equation

One linear and one quadratic equation

- Rearrange the linear equation to make either x or y the subject, then substitute this expression into the other equation.
- Simplify and solve the resulting quadratic equation.
- Use the linear equation to give the values of x or y.

Example: Solve the simultaneous equation $10 - x^2 = y^2$ and $x + y = 2$

$y = 2 - x$	Rearrange the second equation to make x or y the subject
$10 - x^2 = (2 - x)^2$	Sub the linear into the quadratic equation
$10 - x^2 = 4 - 4x + x^2$	Simplify the quadratic, must … = 0
$0 = 2x^2 - 4x - 6$	
$x^2 - 2x - 3 = 0$	Solve the quadratic equation
$(x - 3)(x + 1) = 0$	
$x = 3$ or $x = -1$	
$y = 2 - 3 = -1$ or $y = 2 - (-1) = 3$	Sub values into your rearranged linear equation
$x = 3, y = -1$ or $x = -1, y = 3$	**Must give two pairs of solutions**

MyMaths Reference: Algebra → Equations - quadratic → Quadratic simultaneous equations

I: Inequalities

Linear inequalities

A linear inequality can be treated as a linear equation with one important exception – if you multiply / divide an inequality by a negative quantity, the sign of the inequality reverses.

For example, it is true that 3 > 2 but it would not be true to say – 3 > – 2; this is why the sign must be reversed.

Example:

- Solve $3x+2 \geq 4-x$
  - Remember, if you begin with an 'or equals to' inequality (≤ or ≥) then you must end up with an 'or equals to' inequality, not a strict inequality (< or >).
  - Hopefully, you will not need reminding that a question involving an inequality NEVER EVER ends with x equals…
  - $3x+2 \geq 4-x$

    $4x+2 \geq 4$

    $4x \geq 2$

    $x \geq \frac{1}{2}$

    Not $x = \frac{1}{2}$

MyMaths Reference: Algebra → Inequalities → Inequations

J: <u>Surds</u>

At AS level questions often require exact answers, and so you will need to work confidently with surds.

Simplification of surds

Express as a single surd.
$2\sqrt{5} = \sqrt{2^2} \times \sqrt{5} = \sqrt{2^2 \times 5} = \sqrt{4 \times 5} = \sqrt{20}$

Express as a surd in its simplest form.
$\sqrt{45} = \sqrt{9 \times 5} = \sqrt{9} \times \sqrt{5} = 3\sqrt{5}$

And then you can use this to add/subtract surds.
$\sqrt{50} + \sqrt{98} - \sqrt{8} = 5\sqrt{2} + 7\sqrt{2} - 2\sqrt{2} = 10\sqrt{2}$

Multiplying surds

It is important to remember that $\left(\sqrt{7}\right)^2 = 7.$ Multiplying surds is similar to multiplying brackets. Use the same techniques.

<u>Examples</u>:

- $\sqrt{5}\left(2 + 3\sqrt{5}\right) = 2\sqrt{5} + 3\sqrt{5} \times \sqrt{5} = 2\sqrt{5} + 15$
- $\left(1 + \sqrt{3}\right)\left(3 + 2\sqrt{3}\right) = 3 + 2\sqrt{3} + 3\sqrt{3} + (2 \times 3) = 9 + 5\sqrt{3}$
- $\left(3 - 2\sqrt{5}\right)\left(4 + 3\sqrt{5}\right) = 12 + 9\sqrt{5} - 8\sqrt{5} - (6 \times 5) = \sqrt{5} - 18$

Rationalising a denominator

This means make the denominator into a rational number. At AS level this becomes trickier but you should remember that you can multiply both numerator and denominator by the same surd to achieve this in simple cases.

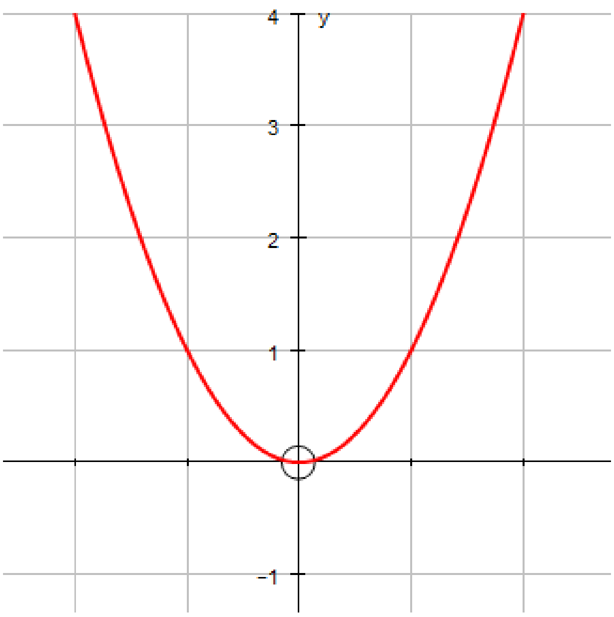
Eg. $\frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

MyMaths Reference: Number → Powers and roots → Surds 1

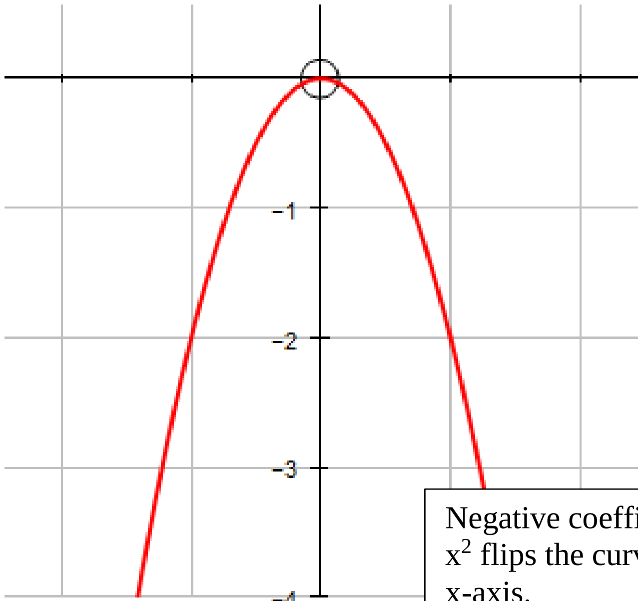
MyMaths Reference: Number → Powers and roots → Surds 2 Q1 only

K: Recognise common graphs – Quadratics, Cubics, Reciprocals

Quadratics



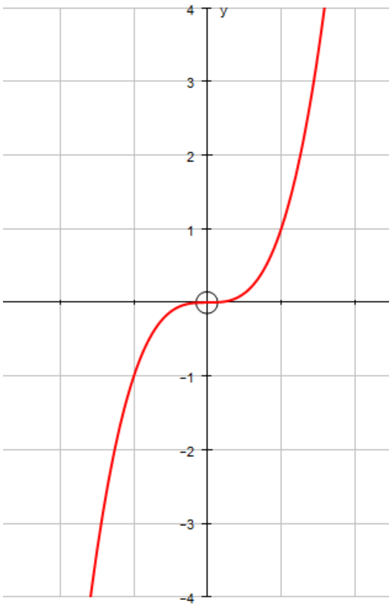
$y = x^2$

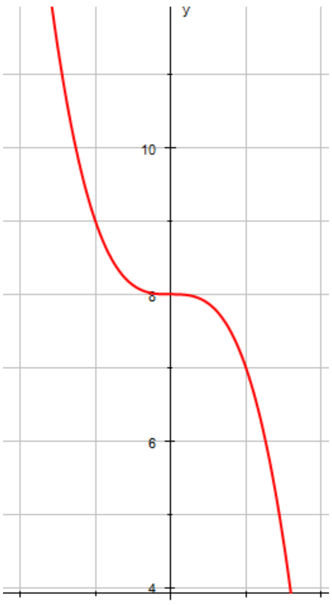


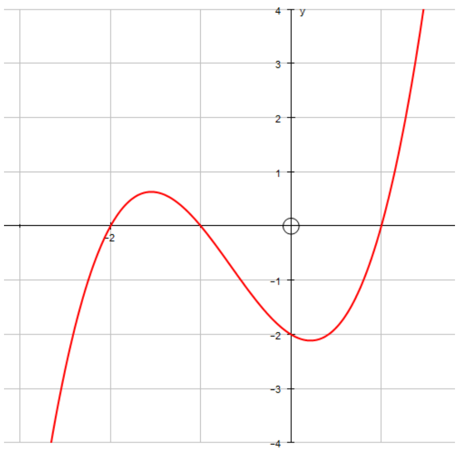
Negative coefficient of x^2 flips the curve in the x-axis.

$y = -2x^2$

Cubics







$y = x^3$

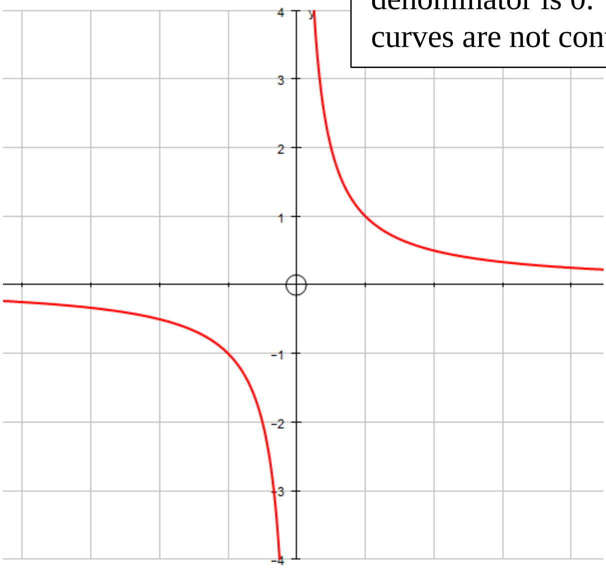
$y = 8 - x^3$

Negative coefficient of x^3

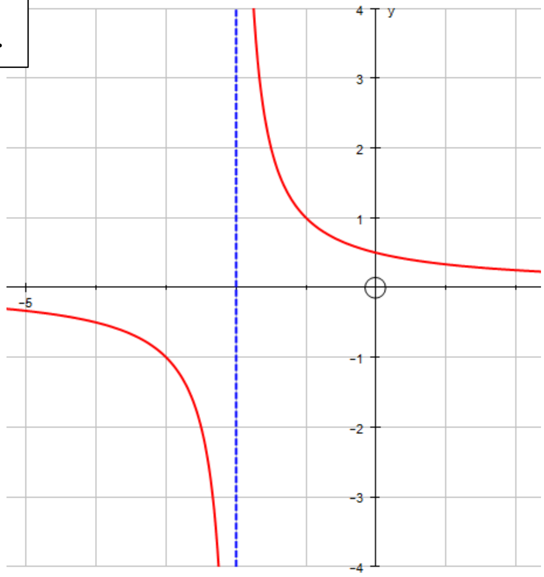
$y = x^3 + 2x^2 - x - 2$

Reciprocals

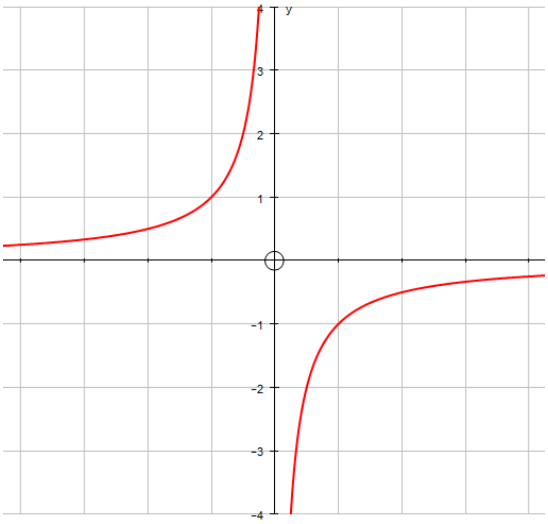
No value when the denominator is 0. These curves are not continuous.



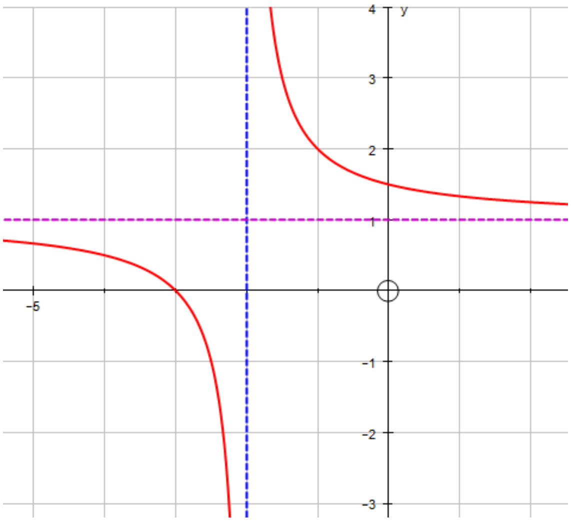
$$y = \frac{1}{x}$$



$$y = \frac{1}{x+2}$$



$$y = -\frac{1}{x}$$



$$y = 1 + \frac{1}{2+x}$$

Asymptotes at x = –2 (because denominator is then zero) and at y = 1 (because when x becomes very large y gets close to 1)

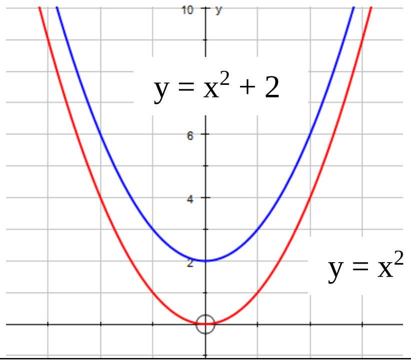
MyMaths Reference: Algebra → Graphs → Recognising graphs

MyMaths Reference: Algebra → Graphs → Reciprocals

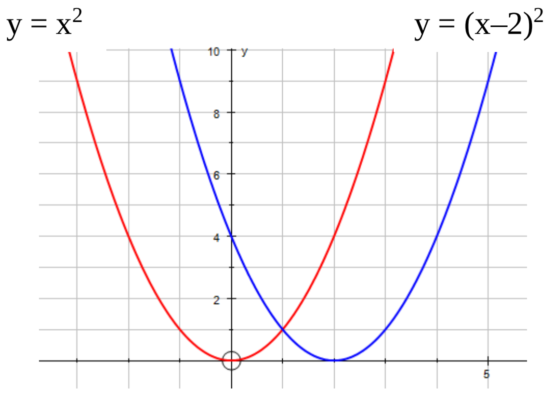
L: Transformation of curves

The work covered in C1 is very similar to the GCSE work, but the language needs to be more mathematical! You will no longer be able to describe a transformation as a move/shift/slide/turn etc but must use the technical language accurately. Remember these transformations on $y = f(x)$.

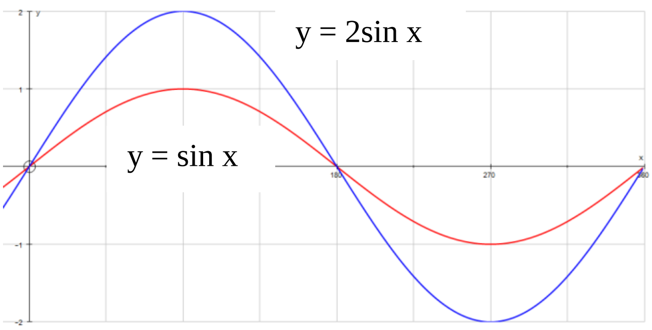
- $y = f(x) + a$ A translation by a units in the positive y-direction (ie moves up by a)
- $y = f(x + a)$ A translation by a units in the negative x- direction (ie moves left by a)
- $y = af(x)$ A stretch parallel to the y axis, by a scale factor of a
- $y = f(ax)$ A stretch parallel to the x axis, by a scale factor of ${}^{1}/_{a}$
- $y = -f(x)$ A reflection in the x axis
- $y = f(-x)$ A reflection in the y axis



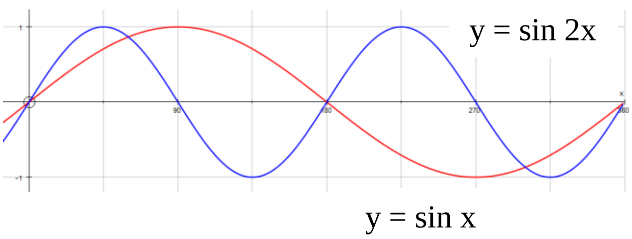
y = f(x) + 2 Translation by 2 units in the positive y-direction



y = f(x – 2) Translation by 2 units in the positive x-direction



y = 2 f(x) Stretch, scale factor 2, parallel to the y-axis.
All points are now twice as far from the x-axis.



y = f(2x) Stretch, scale factor $\frac{1}{2}$, parallel to the x-axis.
All points are now half the distance from the y-axis.
(Remember the curve y = sin x continues on forever)

MyMaths Reference: Algebra → Graphs → Transforming graphs

M: Coordinate Geometry

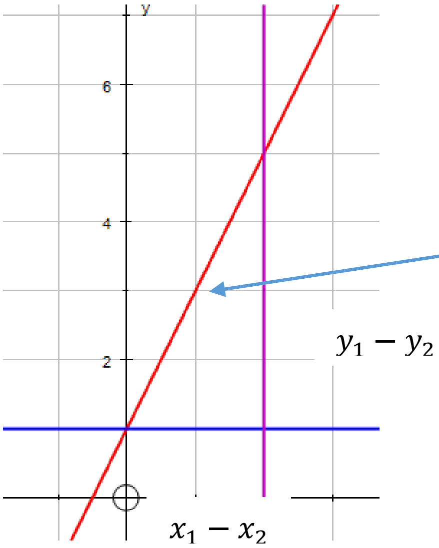
You have met these techniques at GCSE and need to be able to apply these confidently at AS level.

Consider two points A (x_1, y_1) and B (x_2, y_2). Learn these general results.

Midpoint of A and B is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Mean average of the coordinates

Distance between A and B is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$



Pythagoras' Theorem used to work out the length of the hypotenuse.

Gradient of line is change in y divided by change in x.

Gradient of the line joining A and B is $\frac{y_1-y_2}{x_1-x_2} = \frac{y_2-y_1}{x_2-x_1}$

Be consistent in subtracting one set of coordinates from the other.

Let the point A have coordinates (4, –5) and point B have coordinates (–2, 3).

Midpoint is at $\left(\frac{4+(-2)}{2}, \frac{(-5)+3}{2}\right) = (1, -1)$

Distance between A and B is $\sqrt{(4-(-2))^2 + \left((-5)-3\right)^2} = \sqrt{36+64} = 10$

Gradient of the line joining A and B is $m = \frac{(-5)-3}{4-(-2)} = \frac{-8}{6} = -\frac{4}{3}$

(Pay close attention to negative signs!)

MyMaths Reference: Algebra → Coordinates → Midpoint and line length 2

N: Straight Lines

An important topic in its own right at AS as this is the start of Coordinate Geometry. However, these techniques are widely used in other parts of the syllabus too.

The equation of a straight line is given by $y = mx + c$ where m is the gradient and c is the y-intercept. Often at AS the equation of the line is asked for in the format $ax + by = c$ with a, b and c all integers. You must read the question carefully as you will lose the last accuracy mark if you give an equation of the wrong type.

Find the gradient and y-intercept of these straight lines.

$y = 5x - 2$ Gradient = 5 y-intercept = –2

$x + y = 4$ gradient = –1 y – intercept = 4

$3x + 4y = 8$ gradient = $\frac{-3}{4}$ y-intercept = 2

$x + y = 4$
This can be rearranged to give $y = -x + 4$ and remember this is the same as $y = -1x + 4$

***Lines are parallel if they have the same gradient*.**

Two lines are perpendicular if the product of their gradients is –1, i.e. one gradient is the negative reciprocal of the other.

$$m_2 = -\frac{1}{m_1}$$

If $m_1 = \frac{3}{5}$ then the perpendicular gradient $m_2 = -\frac{5}{3}$

Which of these lines are parallel? Which are perpendicular?

A: $y = 2x + 5$ B: $y = -2x + 4$ C: $y = 2x + 7$ D: $2x + y - 9 = 0$

Parallel:	A and C	B and D		
Perpendicular:	A and E	C and E	B and F	D and F

To find the equation of a line given one point on the line and its gradient.
Consider the equation of a line passing through A (x_1, y_1) with gradient m.
Let P (x,y) be any point on the line.

Gradient m = $\frac{y \ - y_2}{x \ - x_2}$

So y - y_1 = m (x - x_1) It is usually easier and quicker to use this rather than finding 'c' to put into y=mx+c

Example
Find the equation of the line passing through (3, -4) and (-1, -2).
Gradient m = $\frac{y_1 - y_2}{x_1 - x_2} = \frac{-2-(-4)}{-1-3} = \frac{2}{-4} = \frac{-1}{2}$
So the equation is
y-(-4) = $\frac{-1}{2}$ (x – 3)
2y + 8 = 3 – x
i.e. 2y + x + 5 = 0

MyMaths Reference: Algebra → Graphs → y = mx + c

MyMaths Reference: Algebra → Graphs → Equation of a line 2

O: Quadratic Inequalities

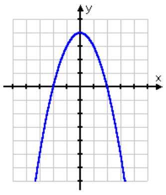
Solve $-x^2 + 4 < 0$.

First, look at the associated two-variable equation, $y = -x^2 + 4$, and consider where its graph is below the x-axis. To do this, you need to know where the graph crosses the x-axis. That is, find where $-x^2 + 4$ is equal to zero:

$-x^2 + 4 = 0$
$x^2 - 4 = 0$
$(x + 2)(x - 2) = 0$
$x = -2$ or $x = 2$

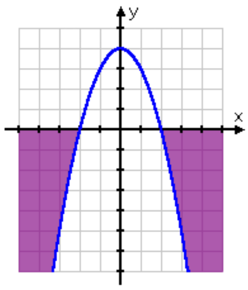
Now figure out where (that is, on which intervals) the graph is below the axis. Since this is a "negative" quadratic, it graphs as an upside-down parabola.

In other words, the graph is high (above the axis) in the middle, and low (below the axis) on the ends:



To solve the original inequality, I need to find the intervals where the graph is below the axis (so the y-values are less than zero).

My knowledge of graphing, together with the zeroes I found above, tells me that that I want the intervals on either end, rather than the interval in the middle:



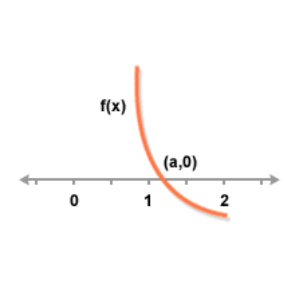
Then the solution is clearly:

$x < -2$ or $x > 2$

MyMaths Reference:
A2A* → Inequalities → Quadratic Inequalities

P: Iteration

Finding approximate roots of polynomial equations.



In the above diagram, a is the root of the function. That is, it's the point where the graph cuts the x axis. The value of $f(a)$ is zero.

We know that:

- The value of $f(1)$ will be *positive* as the curve is *above* the x axis.
- The value of $f(2)$ will be *negative* as the curve is *below* the x axis.

When calculating a root between two given values you must first substitute the two values into the function. If one answer is negative and one positive, then the graph crosses the x axis at a point between the two values given. This is the key to finding the root.

You then have to use trial and error to find the root to the required degree of accuracy.Example

Prove that $x^3 - 4x + 2 = 0$ has a real root between $x = 1$ and $x = 2$ and hence find this root to 1 decimal place.

$f(x) = x^3 - 4x + 2$

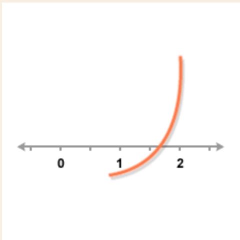
Substitute $x = 1$ and $x = 2$ into the equation

$$f(1) = 1^3 - 4(1) + 2$$
$$= -1$$

$f(1) =$ negative, so it lies below the axis at this point

$$f(2) = 2^3 - 4(2) + 2$$
$$= 2$$

$f(2) =$ positive so it lies above the axis at this point



Graph showing root between 1 and 2

Therefore the root is between $x = 1$ and $x = 2$

Now, let's try $x = 1.5$

$$f(1.5) = 1.5^3 - 4(1.5) + 2$$
$$= -0.625$$

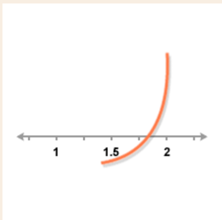
$f(1.5)$ lies below the axis

The root will always be between a positive and a negative value of $f(x)$. So far, we know that:

$f(1) =$ negative

$f(1.5) =$ negative

$f(2) =$ positive



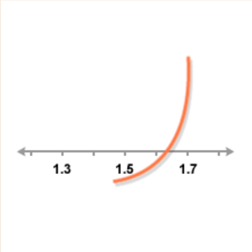
Graph showing root between 1.5 and 2

So the root must lie between $x = 1.5$ and $x = 2$

Try $f(1.7)$

$$f(1.7) = 1.7^3 - 4(1.7) + 2$$
$$= 0.113$$

$f(1.5) =$ negative
$f(1.7) =$ positive
$f(2) =$ positive



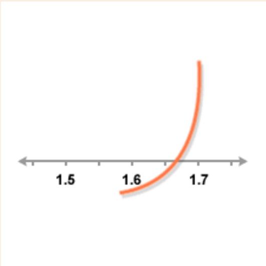
Graph showing root between 1.5 and 1.7

The root is between $x = 1.5$ and $x = 1.7$

Try $f(1.6)$

$$f(1.6) = 1.6^3 - 4(1.6) + 2$$
$$= -0.304$$

$f(1.5) =$ negative
$f(1.6) =$ negative
$f(1.7) =$ positive



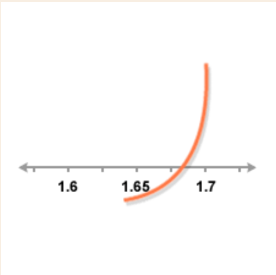
Graph showing root between 1.6 and 1.7

The root is between $x = 1.6$ and $x = 1.7$

Try $f(1.65)$

$$f(1.65) = 1.65^3 - 4(1.65) + 2$$
$$= -0.107875$$

$f(1.6) =$ negative
$f(1.65) =$ negative
$f(1.7) =$ positive



Graph of root between 1.65 and 1.7

The root is between $x = 1.65$ and $x = 1.7$, Hence the root to one decimal place is $x = 1.7$

The last calculation was necessary to work out whether the root lay closer to 1.6 or 1.7.
As you can see from the diagrams, we're zooming in on the root to the desired accuracy.

MyMaths Reference:
Algebra → Equations – approximate solutions → Iterative methods

Q: Inverse functions

This reverses the process of f(x) and takes you back to your original values.
You write the inverse of f(x) as $f^{-1}(x)$.

Example

If $f(x) = 7x - 2$, find $f^{-1}(x)$.

First, rearrange in terms of x :

- $y = 7x - 2$
- $7x - 2 = y$
- $7x = y + 2$
- $x = \frac{y+2}{7}$

Remember to change y back to x when you're writing your answer, thus:

$$f^{-1}(x) = \frac{x+2}{7}$$

Composite functions

Sometimes we are asked to find the result of a function of a function. That is, replacing x with another function.

Follow this worked example.

- $f(x) = 10x + 7$
- $g(x) = 3x$
- Find $f(g(x))$ [sometimes written as fg(x)]
- Replace x with the function
- $f(g(x)) = 10(g(x)) + 7$
- $f(3x) = 10(3x) + 7$

$f(g(x)) = 30x + 7$

Example

$g(x) = 4x^2 + 8x - 7$, $f(x) = x + 1$

Find $g(f(x))$

Answer

Start with g(x) then replace x with the function f(x)

$g(x) = 4x^2 + 8x - 7$

$g(f(x)) = 4(f(x))^2 + 8(f(x)) - 7$

Simplify

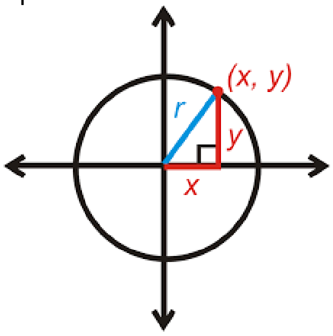
$$\begin{aligned} g(x+1) &= 4(x+1)^2 + 8(x+1) - 7 \\ g(x+1) &= 4(x^2 + 2x + 1) + 8x + 8 - 7 \\ g(f(x)) &= 4x^2 + 8x + 4 + 8x + 1 \\ g(f(x)) &= 4x^2 + 16x + 5 \end{aligned}$$

MyMaths Reference: IGCSE → Functions → Functions and Inverses

R: Circle Equations

Circle: The set of all points on a plane that are a fixed distance from a centre.

Let us put a circle of radius r on a graph:

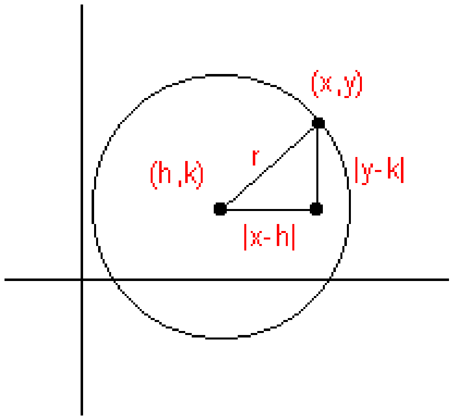


To work out **exactly** where all the points are, we make a right-angled triangle and then use Pythagoras:

$$x^2 + y^2 = r^2$$

More General Case

Now let us put the centre at **(h,k)**



So the circle is **all the points (x,y)** that are **"r"** away from the centre **(h,k)**.

Now lets work out where the points are (using a right-angled triangle and Pythagoras). It is the same idea as before, but we need to subtract **h** and **k**:

$$(x-h)^2 + (y-k)^2 = r^2$$

This is the **"Standard Form"** for the equation of a circle.

(This can also be written as $x^2 + y^2 - 2hx - 2hy + h^2 + k^2 = r^2$)

The standard form shows all the important information at a glance: the centre **(h,k)** and the radius **r**.

Example: A circle with centre at (3,4) and a radius of 6:

Start with: $(x-h)^2 + (y-k)^2 = r^2$

Put in (h,k) and r: $(x-3)^2 + (y-4)^2 = 6^2$

We can then simplify and rearrange that equation, if necessary, depending on what we need it for.

Circle equations are often given in the general format of $ax^2 + by^2 + cx + dy + e = 0$. When you are given this general form of equation and told to find the centre and radius of a circle, you will have to "complete the square" to convert the equation to centre-radius form. This section explains how to make that conversion.

Find the centre and radius of the circle having the following equation:

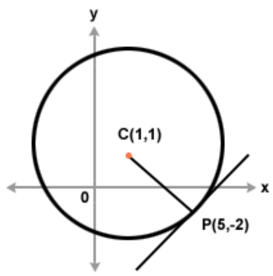
$$4x^2 + 4y^2 - 16x - 24y + 51 = 0.$$

Here is the equation they've given you.	$4x^2 + 4y^2 - 16x - 24y + 51 = 0$
Move the loose number over to the other side.	$4x^2 + 4y^2 - 16x - 24y = -51$
Group the *x*-stuff together. Group the *y*-stuff together.	$4x^2 - 16x + 4y^2 - 24y = -51$
Whatever is multiplied on the squared terms (it'll always be the same number), divide it off from every term.	$x^2 - 4x + y^2 - 6y = -\frac{51}{4}$
This is the complicated step. You'll need space inside your groupings, because this is where you'll add the squaring term. Take the *x*-term coefficient, multiply it by one-half, square it, and then add this to both sides of the equation, as shown. Do the same with the *y*-term coefficient. Convert the left side to squared form, and simplify the right side.	$(x^2 - 4x \quad) + (y^2 - 6y \quad) = -\frac{51}{4}$ $-2 \rightarrow +4 \qquad -3 \rightarrow +9$ $(x^2 - 4x + 4) + (y^2 - 6y + 9) = -\frac{51}{4} + 4 + 9$ $(x - 2)^2 + (y - 3)^2 = \frac{1}{4}$
Read off the answer from the rearranged equation.	The centre is at $(h, k) = (x, y) = (2, 3)$. The radius is $r = \text{sqrt}(\,^1/_4\,) = \,^1/_2$

Finding the Equation of a Tangent to a Circle

Because a tangent is a straight line, you need both a point and the gradient to find its equation. You are usually *given* the point - it's where the tangent meets the circle.

The method for finding the gradient uses the fact that the tangent is perpendicular to the radius from the point it meets the circle. Work out the gradient of the radius at the point the tangent meets the circle, and you can use the equation $m_{CP} \times m_{tgt} = -1$ to find the gradient of the tangent.



Example

Find the equation of the tangent to the circle $x^2 + y^2 - 2x - 2y - 23 = 0$ at the point P(5, -2) which lies on the circle.

- centre = (1,1)
- $m_{CP} = \frac{-2-1}{5-1} = -\frac{3}{4}$
- hence $m_{tgt} = \frac{4}{3}$ since $m_{CP} \times m_{tgt} = -1$

so equation of the tangent at P is

$y - y_1 = m(x - x_1)$

$y - (-2) = \frac{4}{3}(x - 5)$

$3(y + 2) = 4(x - 5)$

$3y - 4x + 26 = 0$,

or

$4x - 3y - 26 = 0$

MyMaths Reference: A level → Core 1 - Graphs → Equations of Circles

MyMaths Reference: Algebra → Graphs → Equation of a line 2

S: Differentiation

Introduction

Differentiation is a technique used to calculate the gradient, or slope, of a graph at different points.

The gradient function

Given a function, for example, $y = x^2$, it is possible to derive a formula for the gradient of its graph. We can think of this formula as the gradient function, precisely because it tells us the gradient of the graph. For example,

when $y = x^2$ the gradient function is $2x$

So, the gradient of the graph of $y = x^2$ at any point is twice the x value there. (An explanation of how this formula is actually found will be covered during you're a level studies). The important point is that using this formula we can calculate the gradient of $y = x^2$ at different points on the graph. For example,

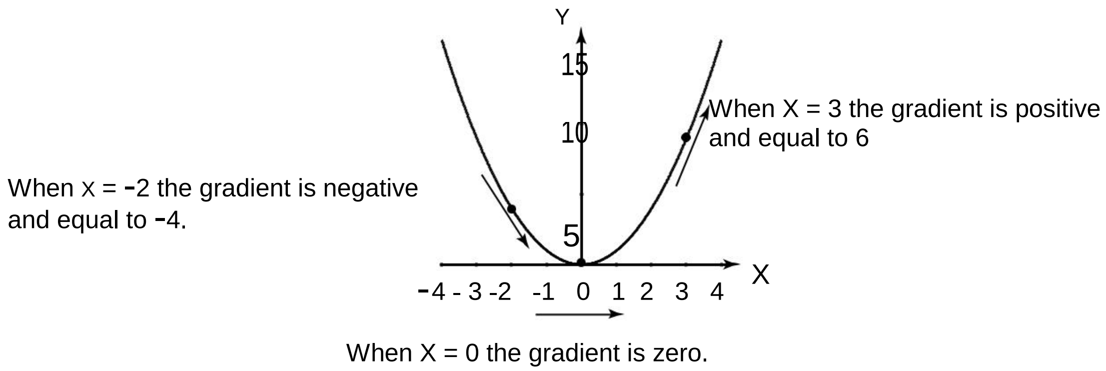
when $x = 3$, the gradient is $2 \times 3 = 6$.

when $x = -2$, the gradient is $2 \times (-2) = -4$.

How do we interpret these numbers ? A gradient of 6 means that values of y are increasing at the rate of 6 units for every 1 unit increase in x. A gradient of -4 means that values of y are decreasing at a rate of 4 units for every 1 unit increase in x.

Note that when $x = 0$, the gradient is $2 \times 0 = 0$.

Below is a graph of the function $y = x^2$. Study the graph and you will note that when $x = 3$ the graph has a positive gradient. When $x = -2$ the graph has a negative gradient. When $x = 0$ the gradient of the graph is zero. Note how these properties of the graph can be predicted from knowledge of the gradient function, $2x$.



Example
When $y = x^3$, its gradient function is $3x^2$.
Calculate the gradient of the graph of $y = x^3$
when
a) $x = 2$,
b) $x = -1$,
c) $x = 0$.

Solution

a) when $x = 2$ the gradient function is $3(2)^2 = 12$.

b) when $x = -1$ the gradient function is $3(-1)^2 = 3$.

c) when $x = 0$ the gradient function is $3(0)^2 = 0$.

Notation for the gradient function

You will need to use a notation for the gradient function which is in widespread use.

If y is a function of x (i.e. y = f(x)), we write its gradient function (or derivative) as $\frac{dy}{\partial x}$
It is pronounced 'dee y by dee x' and is not a fraction even though it might look like one!
Think of $\frac{dy}{\partial x}$ as the 'symbol' for the gradient function of y = f(x).
The process of finding $\frac{dy}{\partial x}$ is a part of calculus called 'differentiationwith respect to x'.

Differentiation Formula

For any value of n, the gradient function of x^n is nx^{n-1}

We write: **if $y = x^n$ then $\frac{dy}{\partial x} = nx^{n-1}$**

(As stated above, an explanation of how this formula is derived will be covered during your A level studies.)

Examples

If $y = x^3$, $\frac{dy}{\partial x} = 3x^2$

If $y = 4x$, $\frac{dy}{\partial x} = 4$

If $y = 5$, $\frac{dy}{\partial x} = 0$

If $y = x^3 + 4x + 5$, $\frac{dy}{\partial x} = 3x^2 + 4$

Find the gradient of $y = x^2$ when $x = 7$

If $y = x^2$, $\frac{dy}{\partial x} = 2x$

At $x = 7$, $\frac{dy}{\partial x} = 2(7) = 14$

Find the gradient of $y = 4x^2 + 5x + 6$ when $x = 3$

If $y = 4x^2 + 5x + 6$, $\frac{dy}{\partial x} = 8x + 5$

At $x = 3$, $\frac{dy}{\partial x} = 8(3) + 5 = 29$

MyMaths Reference: A level → Core 1 - Differentiation → Gradient of a tangent

Assessment questions

A: Expanding brackets: Expand and simplify the following.

1. $(x+2)(x-5)$ 2. $(3x-2y)(2x+y)$ 3. $(2x+1)(2x-1)$ 4. $(a+3b)^2$
5. $(4x-7)(2x+3)$ 6. $(4x-7)(3x-1)$ 7. $(a-b+c)(2a+b-c)$ 8. $(5x-9)^2$
9. $(x^2+7)(x^2-7)$ 10. $(2x^2+x+2)(x^2-3x-4)$

B: Factorising expressions: Factorise the following expressions.

1. x^2+5x+6 2. $x^2-8x-20$ 3. $t^2+5t-36$ 4. $2x^2+11x+15$
5. $5x^2-17x+6$ 6. $3x^2-7x-6$ 7. y^2-64 8. $15+x-2x^2$
9. $5x^2-125$ 10. $6x^2-19x+10$

C: Quadratic equations: Solve the following equations. Give answers to 3sf where appropriate.

1. $x^2+5x+6=0$ 2. $x^2-8=2x$ 3. $2x^2+x-3=0$ 4. $2x^2-11x+15=0$
5. $6x^2+13x+6=0$ 6. $x^2+7x+2=0$ 7. $2x^2+3x=8$

D: Manipulating formulae: Make the letter in the [bracket] the subject of the formulae:

1. $v^2=u^2+2as$ [a] 2. $s=ut+\frac{1}{2}at^2$ [u] 3. $T=2\pi\sqrt{\frac{l}{g}}$ [l]
4. $S=\frac{n}{2}(2a+(n+1)d)$ [d] 5. $S=\frac{a}{1-r}$ [r] 6. $y=\frac{1-x}{2x-3}$ [x]

E: Indices: Evaluate the following

1. 5^{-3} 2. $16^{\frac{1}{4}}$ 3. $8^{-\frac{2}{3}}$ 4. $3a^{-4}\times 2a^{-2}$

F: Completing the square: Complete the square for the following

1. x^2-8x+9 2. x^2+10x 3. x^2-3x+3 4. $2x^2-12x-3$

G: Algebraic fractions: Simplify the following

1. $\frac{3}{x}+\frac{5}{2x}$ 2. $\frac{4x^2}{2x^2+10x}$ 3. $\frac{x^2-9}{x^2-5x+6}$ 4. $\frac{5ac}{12b}\times\frac{14b^2}{15a^3}$

H: Simultaneous equations: Solve the following equations

1. $2x+5y=11$, $4x+3y=29$
2. $4x-3y=1$, $6x+2y=-5$
3. $y=3x-7$, $y=3-5x$
4. $y=2x$, $y^2+xy=x^2+20$

I: Inequalities: Solve the following

1. $3(1-2x)>2(2x+1)$ 2. $\frac{x}{3}\geq\frac{x}{4}+1$

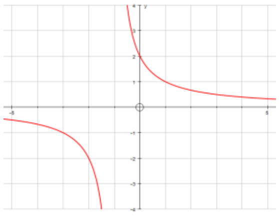
J: Surds: Simplify the following

1. $3\sqrt{5}$ 2. $5\sqrt{2}$ 3. $\sqrt{75}+\sqrt{27}-\sqrt{12}$ 4. $\sqrt{2}(3-5\sqrt{2})$
5. $(2+3\sqrt{2})(1-5\sqrt{2})$
6. Rationalise the denominator $\frac{5}{\sqrt{3}}$

K: Recognise common graphs: Match the equation to its graph.

A $y = 2x - 1$ B $y = x^2 + 2x + 3$ C $y = -x^3 + 6x^2 - 11x + 6$ D $y = \frac{2}{x+1}$

1.



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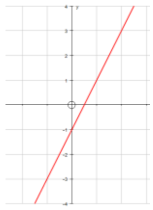
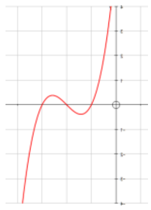


3.



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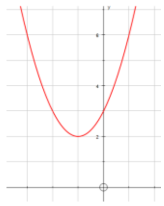

L: Transformations of curves: Describe, in mathematical words, the following transformations.

1. $y = \mathrm{f}(x) + 3$ 2. $y = \mathrm{f}(2x)$ 3. $y = 5\mathrm{f}(x)$ 4. $y = \mathrm{f}(-x)$ 5. $y = \mathrm{f}(x + 6)$ 6. $y = 3\mathrm{f}(x) - 2$

M: Coordinate Geometry: Find the midpoint of AB, the distance AB and the gradient of the line AB.

1. A (3,1) & B (5, 9) 2. A (5, –2) & B (–1, –17) 3. A (8, 14) & B (5, 2)

N: Straight lines: Find the gradient and y-intercept of the following lines.

1. $y = 3 - 2x$ 2. $2x + 3y = 7$ 3. $x + 5y + 9 = 0$ 4. $2x - 3y - 7 = 0$

5. Find the equation of the line passing through the point (4, -2) and perpendicular to a line whose equation is $2x - y - 5 = 0$.

6. The points A and B have co-ordinates (h, k) and (3h, -5k). Find the equation of the perpendicular bisector of AB.

O: Quadratic Inequalities: Solve these inequalities

1. $6 + x - x^2 < 0$ 2. $x^2 - 4x - 5 > 0$ 3. $x^2 - 3x + 2 < 0$ 4. $5x - x^2 > 4$

P: Iteration:

a Show that $x^3 - 9x + 1$ has a root between 0 and 1.

b Show that $x^3 - 9x + 1$ has a root between 0.111 and 0.112. Hence state a root of $x^3 - 9x + 1$ correct to 2 decimal places

Q: Functions – inverse and composite:

Find the inverse functions of these functions.

1. $f(x) = x + 4$ 2. $f(x) = x - 5$

Given that $f(x) = 3x + 4$ and $g(x) = x^2$, find:

3. $f(g(2))$ 4. $f(g(x))$

R: Circle equations:

Find the centre and radius of each of these circles.

1. $x^2 + y^2 - 4x - 2y - 4 = 0$ 2. $x^2 + y^2 + 10x - 14y + 10 = 0$

Find the equations of the tangents to these circles at the points given.

3. $(x - 1)^2 + (y + 2)^2 = 13$; (3,1) 4. $2x^2 + 2y^2 - 8x - 5y - 1 = 0$; (1,-1)

S: Differentiation

1. Find the gradient of $y = x^2 - 2x + 1$ when $x = 2$
2. Find the gradient of $y = x^2 + x + 1$ when $x = 0$
3. Find the gradient of $y = x^2 - 2x$ when $x = -1$
4. Find the gradient of $y = (x + 2)(x - 4)$ when $x = 3$

Assessment solutions

A: Expanding brackets:

1. $x^2-3x-10$ 2. $6x^2-xy-2y^2$ 3. $4x^2-1$ 4. $a^2+6ab+9b^2$
5. $8x^2-2x-21$ 6. $12x^2-25x+7$ 7. $2a^2-ab+ac-b^2+2bc-c^2$ 8. $25x^2-90x+81$
9. x^4-49 10. $2x^4-5x^3-9x^2-10x-8$

B: Factorising expressions:

1. $(x+3)(x+2)$ 2. $(x-10)(x+2)$ 3. $(t+9)(t-4)$ 4. $(2x+5)(x+3)$
5. $(5x-2)(x-3)$ 6. $(3x+2)(x-3)$ 7. $(y+8)(y-8)$ 8. $(2x+5)(3-x)$
9. $5(x+5)(x-5)$ 10. $(3x-2)(2x-5)$

C: Quadratic equations:

1. -3, -2 2. -2, 4 3. $-\frac{3}{2}$, 1 4. $\frac{5}{2}$, 3
5. $-\frac{3}{2}$, $-\frac{2}{3}$ 6. -6.70, -0.298 7. -2.89, 1.39

D: Manipulating formulae:

1. $a=\frac{v^2-u^2}{2s}$ 2. $u=\frac{2s-at^2}{2t}$ 3. $l=\frac{gT^2}{4\pi^2}$
4. $d=\frac{2(S-an)}{n(n+1)}$ 5. $r=\frac{S-a}{S}$ 6. $x=\frac{1+3y}{1+2y}$

E: Indices:

1. $\frac{1}{125}$ 2. 2 3. $\frac{1}{4}$ 4. $6a^{-6}$

F: Completing the square:

1. $(x-4)^2-7$ 2. $(x+5)^2-25$ 3. $\left(x-\frac{3}{2}\right)^2+\frac{3}{4}$ 4. $2(x-3)^2-21$

G: Algebraic fractions:

1. $\frac{11}{2x}$ 2. $\frac{2x}{x+5}$ 3. $\frac{x+3}{x-2}$ 4. $\frac{7bc}{18a^2}$

H: Simultaneous equations:

1. $(8,-1)$ 2. $(-0.5,-1)$ 3. $(1.25,-3.25)$ 4. (2, 4) or (–2, –4)

I: Inequalities:

1. $x<0.1$ 2. $x\geq 12$

J: Surds:

1. $\sqrt{45}$ 2. $\sqrt{50}$ 3. $6\sqrt{3}$ 4. $3\sqrt{2} - 10$ 5. $-28 - 7\sqrt{2}$

6. $\frac{5}{\sqrt{3}} = \frac{5}{3}\sqrt{3}$ or $\frac{5\sqrt{3}}{3}$

K**: Recognise common graphs:**

1. D ($y = \frac{2}{x+1}$) 2. A ($y = 2x - 1$)
3. C ($y = -x^3 + 6x^2 - 11x + 6$) 4. B ($y = x^2 + 2x + 3$)

L: Transformations of curves:

1. $y = \mathrm{f}(x) + 3$ A translation by 3 units in the positive y-direction

2. $y = \mathrm{f}(2x)$ A stretch by scale factor $\frac{1}{2}$ parallel to the x-axis

3. $y = 5\mathrm{f}(x)$ A stretch by scale factor 5 parallel to the y-axis

4. $y = \mathrm{f}(-x)$ A reflection in the y-axis

5. $y = \mathrm{f}(x + 6)$ A translation by –6 units in the positive x-direction

6. $y = 3\mathrm{f}(x) - 2$ A stretch by scale factor 3 parallel to the y-axis, followed by a translation by 2 units in the negative y-direction

M: Coordinate Geometry:

1. midpoint = (4, 5) distance = $\sqrt{68} = 2\sqrt{17}$ gradient = $\frac{9-1}{5-3} = 4$
2. midpoint = $\left(2, \frac{-15}{2}\right)$ distance = $\sqrt{261} = 3\sqrt{29}$ gradient = $\frac{-2-(-17)}{5-(-1)} = \frac{15}{6} = \frac{5}{2}$
3. midpoint = $\left(\frac{13}{2}, 8\right)$ distance = $\sqrt{153} = 3\sqrt{17}$ gradient = $\frac{14-2}{8-5} = \frac{12}{3} = 4$

N: Straight lines:

1. gradient = –2 y-intercept = 3 2. gradient = $-\frac{2}{3}$ y-intercept = $\frac{7}{3}$
3. gradient = $-\frac{1}{5}$ y-intercept = $-\frac{9}{5}$ 4. gradient = $\frac{2}{3}$ y-intercept = $-\frac{7}{3}$
5. x + 2y = 0 6. $hx - 3ky - 6k^2 - 2h^2 = 0$

O: Quadratic Inequalities:

1. x < -2 or x > 3 2. x < -1 or x > 5 3. 1 < x < 2 4. 1 < x < 4

P: Iteration:

b Root is 0.11 (2 d.p.)

Q: Functions – inverse and composite:

1. x - 4 2. x + 5 3. 16 4. $3x^2 + 4$

R: Circle equations:

1. (2,1), 3 2. (-5,7), 8 3. 2x + 3y - 9 = 0 4. 4x + 9y + 5 = 0

S: Differentiation

1. 2 3. 1 4. -4 5. 4